# 7A

(2) (a) 3

(c) 7

(e) 2

(3) $f(3)$

$= (3)^2 - 7(3) + 12$

$= 9 - 21 + 12$

$= 0$

as $f(3) = 0$, $(x-3)$ is a factor.

(4) $f(-4)$

$= (-4)^3 + 2(-4)^2 - 2(-4)$

$= -64 + 32 + 8$

$= -24$

no, $(x+4)$ is not a factor

(6) $f(1)$

$= (1)^3 + 8(1)^2 - (1) - 8$

$= 0$

as $f(1) = 0$, $(x-1)$ is a factor.

(8) $f(1)$

$= 2(1)^5 + 3(1)^2 + 2(1) + 1$

$= 8$

no, $(x-1)$ is not a factor.

# 7B

(1) $x-3$ is a factor $\Rightarrow f(3)=0$

$$\begin{array}{r|rrrr} 3 & 1 & -8 & 19 & -12 \\ & & 3 & -15 & 12 \\ \hline & 1 & -5 & 4 & \boxed{0} \end{array}$$

as $f(3)=0$, $x-3$ is a factor and $x=3$ is a root.

$$f(x) = x^3 - 8x^2 + 19x - 12$$
$$= (x-3)(x^2-5x+4)$$
$$= \underline{\underline{(x-3)(x-1)(x-4)}}$$

(3) $x-2$ is a factor $\Rightarrow f(2)=0$

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 11 & -6 \\ & & 2 & -8 & 6 \\ \hline & 1 & -4 & 3 & \boxed{0} \end{array}$$

$f(2)=0$, $x-2$ is a factor and $x=2$ is a root.

$$f(x) = x^3 - 6x^2 + 11x - 6$$
$$= (x-2)(x^2-4x+3)$$
$$= \underline{\underline{(x-2)(x-1)(x-3)}}$$

7B

(5) $x+3$ is a factor, $f(-3)=0$

$$\begin{array}{r|rrrr} -3 & 1 & 0 & -13 & -12 \\ & & -3 & +9 & 12 \\ \hline & 1 & -3 & -4 & \boxed{0} \end{array}$$

* note we need to use 0 as there is no $x^2$ term.

as $f(-3)=0$, $x+3$ is a factor and $x=-3$ is a root

$x^3 - 13x - 12$

$= (x+3)(x^2-3x-4)$

$= \underline{(x+3)(x+1)(x-4)}$

(8) $x+5$ is a factor so $f(-5)=0$

$$\begin{array}{r|rrrrr} -5 & 1 & 1 & -16 & 20 & 0 \\ & & -5 & 20 & -20 & 0 \\ \hline & 1 & -4 & +4 & 0 & \boxed{0} \end{array}$$

* note no constant (number) term so use 0.

$f(-5)=0$, $\therefore$ $x+5$ is a ~~root~~ factor and $x=-5$ is a root.

$x^3 + x^2 - 16x^2 + 20x$

$= (x+5)(x^3-4x^2+4x) \leftarrow$ common factor of $x$.

$= (x+5)\,x\,(x^2-4x+4)$

$= x(x+5)(x-2)(x-2)$

$= \underline{x(x-2)^2(x+5)}$

# 7C

1(a) $f(1) = (1)^3 - (1)^2 - (1) + 1$
$= 0$

$$\begin{array}{c|cccc}
1 & 1 & -1 & -1 & +1 \\
 & & 1 & 0 & -1 \\
\hline
 & 1 & 0 & -1 & \boxed{0}
\end{array}$$

$f(1) = 0$, $\therefore (x-1)$ is a factor and $x = 1$ is a root.

$f(x) = x^3 - x^2 - x + 1$
$= (x-1)(x^2-1)$
$= (x-1)(x-1)(x+1)$
$= \underline{\underline{(x-1)^2(x+1)}}$

(e) $f(1) = (1)^3 - 2(1)^2 - 4(1) + 8$
$= 3$

$f(-1) = (-1)^3 - 2(-1)^2 - 4(-1) + 8$
$= 9$

$f(2) = (2)^3 - 2(2)^2 - 4(2) + 8$
$= 0$

$$\begin{array}{c|cccc}
2 & 1 & -2 & -4 & 8 \\
 & & 2 & 0 & -8 \\
\hline
 & 1 & 0 & -4 & \boxed{0}
\end{array}$$

$f(2) = 0$, $\therefore x-2$ is a factor and $x = 2$ is a root.

$x^3 - 2x^2 - 4x + 8$
$= (x-2)(x^2-4)$
$= \underline{\underline{(x-2)^2(x+2)}}$

# 7C

2(c) $f(1) = (1)^3 - 4(1)^2 + (1) + 6$

$= 4$

$f(-1) = (-1)^3 - 4(-1)^2 + (-1) + 6$

$= 0$

$$\begin{array}{r|rrrr} -1 & 1 & -4 & 1 & 6 \\ & & -1 & 5 & -6 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

$f(-1) = 0$, $\therefore$ $(x+1)$ is a factor and $x = -1$ is a root.

$x^3 - 4x^2 + x + 6$

$= (x+1)(x^2 - 5x + 6)$

$= \underline{\underline{(x+1)(x-2)(x-3)}}$

2(f) $f(1) = (1)^3 - 19(1) + 30$

$= 12$

$f(-1) = (-1)^3 - 19(-1) + 30$

$= 48$

$f(2) = (2)^3 - 19(2) + 30$

$= 0$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -19 & 30 \\ & & 2 & 4 & -30 \\ \hline & 1 & 2 & -15 & \boxed{0} \end{array}$$

$\boxed{\text{* no } x^2 \text{ term !}}$

$f(2) = 0$, $\therefore$ $(x-2)$ is a factor and $x = 2$ is a root.

$(x-2)(x^2 + 2x - 15)$

$= \underline{\underline{(x-2)(x+5)(x-3)}}$

# 7C

3(b) $f(1) = 0$

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 0 | -1 | 0 |
| | | 1 | 1 | 0 |
| | 1 | 1 | 0 | $\boxed{0}$ |

$f(1) = 0$, $\therefore$ $(x-1)$ is a factor and $x = 1$ is a root

$x^3 - x$

$= (x-1)(x^2 + x)$

$= (x-1)x(x+1)$

$= \underline{\underline{x(x-1)(x+1)}}$

Note: we could factorise this without synthetic division

$x^3 - x$

$= x(x^2 - 1)$

$= \underline{\underline{x(x-1)(x+1)}}$

3(c) $f(1)$

$= (1)^4 - 3(1)^3 - 6(1)^2 + 8(1)$

$= 0$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | -3 | -6 | 8 | 0 |
| | | 1 | -2 | -8 | 0 |
| | 1 | -2 | -8 | 0 | $\boxed{0}$ |

$f(1) = 0$, $\therefore$ $x - 1$ is a factor and $x = 1$ is a root.

$x^4 - 3x^3 - 6x^2 + 8x$

$= (x-1)(x^3 - 2x^2 - 8x)$

$= (x-1)x(x^2 - 2x - 8)$

$= x(x-1)(x-4)(x+2)$

## 7D

(1) (a) $2x-1$ is a factor

$$2x-1=0$$
$$2x=1$$
$x=\frac{1}{2}$ is a root

$$\begin{array}{c|cccc}
\frac{1}{2} & 2 & 11 & 4 & -5 \\
 & & 1 & 6 & 5 \\
\hline
 & 2 & 12 & 10 & \boxed{0}
\end{array}$$

$f(\frac{1}{2})=0$, $(x-\frac{1}{2})$ is a factor, and $x=\frac{1}{2}$ is a root. As $(x-\frac{1}{2})$ is a factor $(2x-1)$ will also be a factor.

(b) $f(x)=(x-\frac{1}{2})(2x^2+12x+10)$

$$=(x-\tfrac{1}{2})\,2(x^2+6x+5)$$
$$=2(x-\tfrac{1}{2})(x^2+6x+5)$$
$$=(2x-1)(x^2+6x+5)$$
$$=\underline{\underline{(2x-1)(x+5)(x+1)}}$$

7D

(2) (c) $2x^3 - 11x^2 + 17x - 6$

$f(1) = 2(1)^3 - 11(1)^2 + 17(1) - 6$

$= 2$

$f(-1) = 2(-1)^3 - 11(-1)^2 + 17(-1) - 6$

$= -36$

$f(2) = 2(2)^3 - 11(2)^2 + 17(2) - 6$

$= 0$

$$\begin{array}{c|cccc} 2 & 2 & -11 & 17 & -6 \\ & & 4 & -14 & 6 \\ \hline & 2 & -7 & 3 & \boxed{0} \end{array}$$

$f(2) = 0$, $x = 2$ is a root and $(x-2)$ is a factor

$2x^3 - 11x^2 + 17x - 6$

$= (x-2)(2x^2 - 7x + 3)$

$= \underline{\underline{(x-2)(2x-1)(x-3)}}$

$=$

7D

2(h) $f(1) = 6(1)^3 + 11(1)^2 - 3(1) - 2$

$= 12$

$f(-1) = 6(-1)^3 + 11(-1)^2 - 3(-1) - 2$

$= 6$

$f(2) = 6(2)^3 + 11(2)^2 - 3(2) - 2$

$= 84$

$f(-2) = 6(-2)^3 + 11(-2)^2 - 3(-2) - 2$

$= 0$

| -2 | 6 | 11 | -3 | -2 |
|---|---|---|---|---|
| | | -12 | 2 | 2 |
| | 6 | -1 | -1 | 0 |

$f(-2) = 0$, $x = -2$ is a root and $x+2$ is a factor.

$6x^3 + 11x^2 - 3x - 2$

$= (x+2)(6x^2 - x - 1)$

$= \underline{\underline{(x+2)(3x+1)(2x-1)}}$

7D

3 (e) $f(1) = (1)^4 - (1)^3 - 3(1)^2 + (1) + 2$

$= 0$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | -1 | -3 | 1 | 2 |
| | | 1 | 0 | -3 | -2 |
| | 1 | 0 | -3 | -2 | $\boxed{0}$ |

$f(1) = 0$, $\therefore x = 1$ is a root and $(x-1)$ is a factor.

$(x-1)(x^3 - 3x - 2)$

factorise this cubic using synthetic division

do not forget this factor!

$f(1) = -5$

$f(-1) = 0$

| | | | | |
|---|---|---|---|---|
| -1 | 1 | 0 | -3 | -2 |
| | | -1 | 1 | 2 |
| | +1 | -1 | -2 | $\boxed{0}$ |

$f(-1) = 0$, $x = -1$ is a root and $(x+1)$ is a factor.

$x^4 - x^3 - 3x^2 + x + 2$

$= (x-1)(x+1)(x^2 - x - 2)$

$= (x-1)(x+1)(x-2)(x+1)$

$= \underline{\underline{(x-2)(x-1)(x+1)^2}}$

7D

(4) (c) $x^4 - 5x^2 + 4$

$= (x^2-4)(x^2-1)$

$= (x-2)(x+2)(x-1)(x+1)$

We can solve this by inspection. We could use synthetic division if we did not solve by inspection.

(g) $x^4 - x^2 - 12$

$= (x^2-4)(x^2+3)$

$= (x-2)(x+2)(x^2+3)$

$x^2+3 > 0$ for all values of $x$ i.e. no real roots

so $x^2+3$ cannot be factorised.

(or use $b^2-4ac$ to show no real roots.)

(5) $H = x+1$, $V_{cuboid}$ given by $V = LBH$ i.e. $x+1$ is a factor & $x=-1$ is a root

$$\begin{array}{r|rrrr} -1 & 1 & -10 & 13 & 24 \\ & & -1 & 11 & -24 \\ \hline & 1 & -11 & 24 & \boxed{0} \end{array}$$

$f(-1) = 0$ $x=-1$ is a root and $x+1$ is a factor.

$V(x) = (x+1)(x^2-11x+24)$

$= (x+1)(x-3)(x-8)$

i.e. $H = (x+1)$ cm

$L = (x-3)$ cm

$B = (x-8)$ cm

7D

6 $V(x) = x^3 + x^2 - 8x - 12$

$x-3$ is a factor $\therefore x=3$ is a root.

$$\begin{array}{r|rrrr} 3 & 1 & 1 & -8 & -12 \\ & & 3 & 12 & 12 \\ \hline & 1 & 4 & 4 & \boxed{0} \end{array}$$

$f(3)=0$, $\therefore x=3$ is a root and $x-3$ is a factor.

$V(x) = (x-3)(x^2+4x+4)$

$= \underline{\underline{(x-3)(x+2)(x+2)}}$

Square based cuboid $\therefore B = x+2$, $H = x-3$ as mentioned in question. Difference between B and H $= (x+2)-(x-3)$

$= \underline{\underline{5 \text{ units}}}$

# 7E

① $x-1$ is a factor $\therefore x=1$ is a root, $f(1)=0$

$$\begin{array}{r|rrrr} 1 & 1 & p & 2 & -8 \\ & & 1 & p+1 & p+3 \\ \hline & 1 & p+1 & p+3 & \boxed{p-5} \end{array} = 0 \quad \text{as } x=1 \text{ is a root.}$$

$$p-5=0$$

$$\underline{\underline{p=5}}$$

OR $x=1$ is a root, $f(1)=0$

$$f(1) = (1)^3 + p(1)^2 + 2(1) - 8$$
$$= p-5$$
$$= 0$$

$$p-5=0$$

$$\underline{\underline{p=5}}$$

② $x-2$ is a factor, $\therefore x=2$ is a root

$$\begin{array}{r|rrrr} 2 & 1 & -9 & 24 & -q \\ & & 2 & -14 & 20 \\ \hline & 1 & -7 & 10 & \boxed{20-q} \end{array} = 0 \quad \text{as } x=2 \text{ is a root.}$$

$$20-q=0$$

$$\underline{\underline{q=20}}$$

## 7E

(3) $x - 1$ is a factor, $\therefore x = 1$ is a root.

$$
\begin{array}{r|rrrr}
1 & 2k & k & -2 & -1 \\
 & & 2k & 3k & 3k-2 \\
\hline
 & 2k & 3k & 3k-2 & \boxed{3k-3} = 0
\end{array}
$$

as $x = 1$ is a root.

$$3k - 3 = 0$$
$$3k = 3$$
$$\underline{\underline{k = 1}}$$

$2kx^3 + kx^2 - 2x - 1$

$= 2x^3 + x^2 - 2x - 1$ sub in $k=1$

$= (x-1)(2x^2 + 3x + 1)$

$= \underline{\underline{(x-1)(2x+1)(x+1)}}$

(4) $x + 3$ is a factor, $\therefore x = -3$ is a root.

$$
\begin{array}{r|rrrrr}
-3 & 1 & a & -7 & -43 & 42 \\
 & & -3 & -3a+9 & 9a-6 & -27a+147 \\
\hline
 & 1 & a-3 & -3a+2 & 9a-49 & \boxed{-27a+189} = 0
\end{array}
$$

as $x = -3$ is a root.

$$-27a + 189 = 0$$
$$189 = 27a$$
$$\underline{\underline{a = 7}}$$

$a = 7$

$x^3 + 4x - 19x + 14$

$$
\begin{array}{r|rrrr}
1 & 1 & 4 & -19 & 14 \\
 & & 1 & 5 & -14 \\
\hline
 & 1 & 5 & -14 & \boxed{0}
\end{array}
$$

$x = 1$ is a root, $x - 1$ is a factor

$f(x) = (x+3)(x-1)(x^2 + 5x - 14) = \underline{\underline{(x+3)(x-1)(x+7)(x-2)}}$

factor from start of question.

7E

6 $x+1$ is a factor, $x=-1$ is a root.

$$\begin{array}{r|rrrr} -1 & a & -3 & -3 & b \\ & & -a & a+3 & -a \\ \hline & a & -a-3 & a & \boxed{b-a} \end{array} = 0 \quad \text{as } x=-1 \text{ is a root}$$

$$b-a=0 \quad ①$$

$x-2$ is a factor, $x=2$ is a root

$$\begin{array}{r|rrrr} 2 & a & -3 & -3 & b \\ & & 2a & 4a-6 & 8a-18 \\ \hline & a & 2a-3 & 4a-9 & \boxed{8a+b-18=0} \end{array}$$

$$8a+b=18 \quad ②$$

$$9a = 18 \quad ② - ①$$

$$\underline{\underline{a = 2}}$$

from ① $\underline{\underline{b = 2}}$

7E

7. $x+3$ is a factor, $x=-3$ is a root.

$$
\begin{array}{r|rrrrl}
-3 & 1 & 2 & -7 & p & q \\
 & & -3 & 3 & 12 & -3p-36 \\
\hline
 & 1 & -1 & -4 & p+12 & \boxed{-3p+q-36=0}
\end{array}
$$

$$-3p+q=36 \quad (1)$$

$x-2$ is a factor, $x=2$ is a root.

$$
\begin{array}{r|rrrrl}
2 & 1 & 2 & -7 & p & q \\
 & & 2 & 8 & 2 & 2p+4 \\
\hline
 & 1 & 4 & 1 & p+2 & \boxed{2p+q+4=0}
\end{array}
$$

$$2p+q=-4 \quad (2)$$

$$5p=-40 \quad (2)-(1)$$

$$\underline{\underline{p=-8}}$$

$$2(-8)+q+4=0 \quad (2)$$

$$q-12=0$$

$$\underline{\underline{q=12}}$$

Factorise this using $x+3$ as a factor and $p=-8$

$$
\begin{array}{r|rrrr}
-3 & 1 & 4 & 1 & -6 \\
 & & -3 & -3 & 6 \\
\hline
 & 1 & 1 & -2 & \boxed{0}
\end{array}
$$

$f(-3)=0$, $\therefore$ $x=-3$ is a root and $x+3$ is a factor.

$$f(x)=(x-2)(x+3)(x^2+x-2)$$

$$=\underline{\underline{(x-2)(x+3)(x+2)(x-1)}}$$

## 7F

1(b) Dividing by $(x+2)$ so find ~~x~~ $f(-2)$

$$
\begin{array}{r|rrrr}
-2 & 2 & 1 & -3 & -4 \\
 & & -4 & 6 & -6 \\
\hline
 & 2 & -3 & 3 & \boxed{-10}
\end{array}
$$

$\underline{f(-2) = -10}$ $\therefore$ $\underline{\underline{\text{remainder} = -10}}$

1(d) Dividing by $(2x-1)$ so find $f(\frac{1}{2})$

$$
\begin{array}{r|rrrr}
\frac{1}{2} & 2 & 0 & -8 & 3 \\
 & & 1 & \frac{1}{2} & -\frac{15}{4} \\
\hline
 & 2 & 1 & -\frac{15}{2} & \boxed{-\frac{3}{4}}
\end{array}
$$

$3 - \frac{15}{4}$

$= \frac{12}{4} - \frac{15}{4}$

$= \underline{\underline{\frac{-3}{4}}}$

$\underline{\underline{f(\frac{1}{2}) = -\frac{3}{4}}}$ $\therefore$ $\underline{\underline{\text{remainder} = -\frac{3}{4}}}$

7F

2 (a) $x+1$ has a remainder of 5, $\therefore f(-1) = 5$

$$
\begin{array}{r|rrrl}
-1 & 1 & a & -3 & 1 \\
 & & -1 & -a+1 & a+2 \\
\hline
 & 1 & a-1 & -a-2 & \boxed{a+3} = 5
\end{array}
$$

$$a + 3 = 5$$

$$\underline{\underline{a = 2}}$$

(c) $f\left(\frac{1}{2}\right) = 3$

$$
\begin{array}{r|rrrl}
\frac{1}{2} & a & 0 & 6 & -1 \\
 & & \frac{1}{2}a & \frac{1}{4}a & \frac{1}{8}a+3 \\
\hline
 & a & \frac{1}{2}a & \frac{1}{4}a+6 & \boxed{\frac{1}{8}a+2} = 3
\end{array}
$$

$$\frac{1}{8}a + 2 = 3$$

$$\frac{1}{8}a = 1$$

$$\underline{\underline{a = 8}}$$

## 7F

③ $f(1) = 5, \quad f(-1) = -11$

$$\begin{array}{r|rrrr} 1 & a & -2 & 5 & b \\ & & a & a-2 & a+3 \\ \hline & a & a-2 & a+3 & a+b+3=5 \end{array}$$

$a+b=2$ ①

$$\begin{array}{r|rrrr} -1 & a & -2 & 5 & b \\ & & -a & a+2 & -a-7 \\ \hline & a & -a-2 & a+7 & -a+b-7=-11 \end{array}$$

$-a+b=-4$ ②

$2b=-2$ ① + ②

$\underline{b=-1}$

From ① $\underline{\underline{a=3}}$

④ $f(1)=0 \quad f(2)=-12$

$$\begin{array}{r|rrrrr} 1 & 1 & a & -7 & b & 6 \\ & & 1 & a+1 & a-6 & a+b-6 \\ \hline & 1 & a+1 & a-6 & a+b-6 & a+b=0 \end{array}$$

$a+b=0$ ①

$$\begin{array}{r|rrrrr} 2 & 1 & a & -7 & b & 6 \\ & & 2 & 2a+4 & 4a-6 & \cancel{4a+b-6}\ 8a+2b-12 \\ \hline & 1 & a+2 & 2a-3 & 4a+b-6 & \cancel{4a+b=-12}\ 8a+2b-6=-12 \end{array}$$

$\cancel{4a+b=-12}$ ② $8a+2b=-6$ ②

$\cancel{3a=-12}$ ② $-$ ① $2a+2b=0$ ③ (① ×2)

$\cancel{a=-4}$ $6a=-6$ ② $-$ ③

$\underline{a=-1}$

~~From ①~~ $\cancel{b=4}$ $\Rightarrow \underline{b=1}$

# 7F

(5) (a) find $f(2)$

$$\begin{array}{r|rrrr} 2 & 1 & 5 & -3 & 1 \\ & & 2 & 14 & 22 \\ \hline & 1 & 7 & 11 & \boxed{23} \end{array}$$

Quotient $= x^2 + 7x + 11$

Remainder $= 23$

(b)

(c) find $f(-1)$

$$\begin{array}{r|rrrr} -1 & 2 & -1 & 3 & 6 \\ & & -2 & 3 & -6 \\ \hline & 2 & -3 & 6 & \boxed{0} \end{array}$$

Quotient $= 2x^2 - 3x + 6$

remainder $= 0$ i.e. $x+1$ is a factor of $2x^3 - x^2 + 3x + 6$

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$f(1) = -48$

① (c) $f(-1) = 0$

$$\begin{array}{r|rrrr} -1 & 1 & 1 & -25 & -25 \\ & & -1 & 0 & 25 \\ \hline & 1 & 0 & -25 & \boxed{0} \end{array}$$

$x = -1$ is a root and $x+1$ is a factor

$$y = x^3 + x^2 - 25x - 25$$
$$= (x+1)(x^2 - 25)$$
$$= (x+1)(x-5)(x+5)$$

on $x$-axis $y = 0$

$$0 = (x+1)(x-5)(x+5)$$

$x + 1 = 0 \qquad x - 5 = 0 \qquad x + 5 = 0$

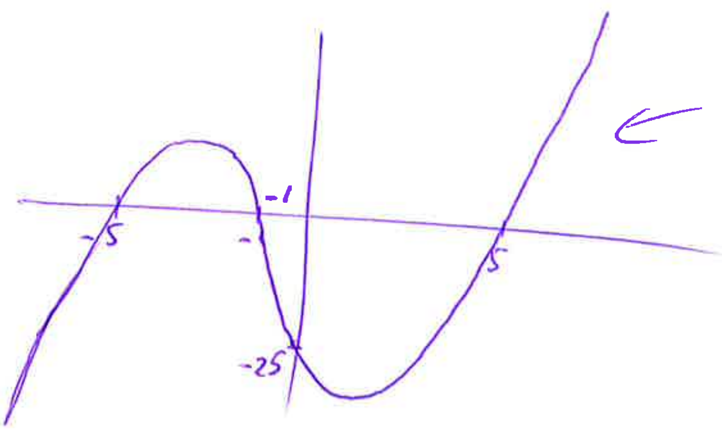
$x = -1 \qquad x = 5 \qquad x = -5$

curve meets $\underline{x\text{-axis}}$ at $\underline{(-5,0), (-1,0), (5,0)}$

on $y$-axis $x = 0$

$f(0) = -25$

curve meets $\underline{y\text{-axis}}$ at $\underline{(0,-25)}$




not asked for

7G ② $P = t^3 - 8t^2 + 15t$ no constant term so factorise (number)

$P(t) = t(t^2 - 8t + 15)$

$P(t) = t(t-3)(t-5)$

zero profit when $P(t) = 0$

$0 = t(t-3)(t-5)$

$t = 0$ $t - 3 = 0$ $t - 5 = 0$

$t = 0$ $t = 3$ $t = 5$

zero profit in months 0, 3 and 5.

7H

1(a) $f(x) = k(x-1)(x-2)(x-3)$

$(0,-6) \Rightarrow x=0, y=-6$

$-6 = k(-1)(-2)(-3)$

$-6 = -6k$

$k = 1$

$f(x) = (x-1)(x-2)(x-3)$

1(c) $f(x) = k(x+2)(x-2)(x-4)$

$(1,27) \Rightarrow x=1, y=27$

$27 = k(1+2)(1-2)(1-4)$

$27 = k(3)(-1)(-3)$

$27 = 9k$

$k = 3$

$f(x) = 3(x+2)(x-2)(x-4)$

7H

1(e) $f(x) = k(x+3)(x-2)^2$

* repeated root as curve touches but does not cross x-axis at $x = 2$

$8 = k(1+3)(1-2)^2$

$8 = 4k$

$k = 2$

$\underline{\underline{f(x) = 2(x+3)(x-2)^2}}$

1(g) $f(x) = k(x+2)(x-1)(x-3)$

$8 = k(2+2)(2-1)(2-3)$

$8 = -4k$

$\underline{k = -2}$

$\underline{\underline{f(x) = -2(x+2)(x-1)(x-3)}}$

7H

1(i) $f(x) = k(x+1)^2(x-1)^2$

$1 = k(0+1)^2(0-1)^2$

$1 = k$

$k = 1$

$f(x) = (x+1)^2(x-1)^2$

* this is a quartic (power 4, $x^4$) graph not a cubic, hence upto 4 roots (in this case 2 repeated roots).

(3) $f(x) = kx(x-25)(x-50)(x-75)$

$12 = k(10)(10-25)(10-50)(10-75)$

$12 = -390\,000k$

$k = \frac{-12}{390\,000}$

$k = \frac{-4}{130\,000}$

$k = \frac{-1}{32\,500}$

$f(x) = \frac{-1}{32\,500}x(x-25)(x-50)(x-75)$

* 4 roots, remember $x=0$ is a root so $x$ is a factor

4 7#

$f(x) = k(x+a)(x-2)(x-3)$

$12 = k(0+a)(0-2)(0-3)$

$12 = 6ak$ ①

$2 = ak$ ①

$8 = k(1+a)(1-2)(1-3)$

$8 = 2k(1+a)$

$4 = k(1+a)$ ②

rearrange ①

$k = 2/a$

sub in ②

$4 = \frac{2}{a}(1+a)$

$4a = 2(1+a)$

$4a = 2 + 2a$

$2a = 2$

$\underline{\underline{a = 1}}$

$k = 2/a$

$k = 2/1$

$\underline{\underline{k = 2}}$

7J

① $(-2,-8)$, $(3,7)$, $(4,10)$

2 (a) $x^3 - 25 = 11 - 7x^2$

$x^3 + 7x^2 - 36 = 0$

$f(1) = -28$

$f(-1) = -30$

$f(2) = 0$

no $x$-term

$$\begin{array}{c|cccc} 2 & 1 & 7 & 0 & -36 \\ & & 2 & 18 & 36 \\ \hline & 1 & 9 & 18 & 0 \end{array}$$

$r=0$ $\therefore$ $x=2$ is a root and $x+2$ is a factor.

$(x-2)(x^2+9x+18)=0$

$(x-2)(x+3)(x+6)=0$

$x=2$, $x=-3$ $x=-6$

$y(2) = 11 - 7(2)^2$
$= -17$

$(2,-17)$

$y(-3) = 11 - 7(-3)^2$
$= -52$

$(-3,-52)$

$y(-6) = 11 - 7(-6)^2$
$= -241$

$(-6,-241)$

7J

2(c) $x^3 + 9x^2 + 3x - 5 = 16 - 8x$

$x^3 + 9x^2 + 11x - 21 = 0$

$$\begin{array}{r|rrrr} 1 & 1 & 9 & 11 & -21 \\ & & 1 & 10 & 21 \\ \hline & 1 & 10 & 21 & 0 \end{array}$$

$r = 0 \quad \therefore x = 1$ is a root and $(x-1)$ is a factor.

$(x-1)(x^2 + 10x + 21) = 0$

$(x-1)(x+3)(x+7) = 0$

$x = 1 \quad x = -3 \quad x = -7$

$y(1) = 16 - 8(1)$
$= 8$

$\underline{\underline{(1, 8)}}$

$y(-3) = 16 - 8(-3)$
$= 40$

$\underline{\underline{(-3, 40)}}$

$y(-7) = 16 - 8(-7)$
$= 72$

$\underline{\underline{(-7, 72)}}$

7J

(3) (b) $y = y$

$$3x^3 + 3x^2 + x - 5 = x^3 - 2x^2 + 2x + 1$$

$$2x^3 + 5x^2 - x - 6 = 0$$

$f(1) = 0$

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 5 | -1 | -6 |
| | | 2 | 7 | 6 |
| | 2 | 7 | 6 | 0 |

as $f(1) = 0$ $x = 1$ is a root and $(x-1)$ is a factor.

$$(x-1)(2x^2 + 7x + 6) = 0$$

$$(x-1)(2x+3)(x+2) = 0$$

$x - 1 = 0$ $\quad$ $2x + 3 = 0$ $\quad$ $x + 2 = 0$

$\underline{\underline{x = 1}}$ $\quad$ $2x = -3$

$\underline{\underline{x = -3/2}}$ $\quad$ $\underline{\underline{x = -2}}$

sub $x = 1$ into either equation

$$y(1) = (1)^3 - 2(1)^2 + 2(1) + 1$$
$$= 2$$

$$y(-\tfrac{3}{2}) = -9.875$$

$$y(-2) = -19$$

$$\underline{\underline{(-2, -19), \left(-\tfrac{3}{2}, -9.875\right), (1, 2)}}$$

7J

$$y = y$$

3(d) $4x^4 + x^3 - x^2 + 1 = x^3 + 4x^2$

$$4x^4 - 5x^2 + 1 = 0$$

$$(4x^2 - 1)(x^2 - 1) = 0$$

$$(2x-1)(2x+1)(x-1)(x+1) = 0$$

• Note we can factorise this by inspection or use synthetic division.

or $4x^4 - 5x^2 + 1 = 0$

$f(1) = 0$

| 1 | 4 | 0 | -5 | 0 | 1 |
|---|---|---|---|---|---|
| | | 4 | 4 | -1 | -1 |
| | 4 | 4 | -1 | -1 | 0 |

$f(1) = 0$ ∴ $x=1$ is a root and $(x-1)$ is a factor

$$(x-1)(4x^3 + 4x^2 - x - 1) = 0$$

$f(-1) = 0$

| -1 | 4 | 4 | -1 | -1 |
|---|---|---|---|---|
| | | -4 | 0 | 1 |
| | +4 | 0 | -1 | 0 |

$f(-1) = 0$ ∴ $x=-1$ is a root and $(x+1)$ is a factor.

$$(x-1)(x+1)(4x^2-1) = 0$$

$$(x-1)(x+1)(2x-1)(2x+1) = 0$$

$$x = 1, -1, \tfrac{1}{2}, -\tfrac{1}{2}$$

$$y(1) = (1)^3 + 4(1)^2 = 5$$

$$y(-1) = 3$$

$y(\tfrac{1}{2}) = 1.125$, $y(-\tfrac{1}{2}) = 0.875$

$(1,5), (-1,3), (\tfrac{1}{2}, 1.125), (-\tfrac{1}{2}, 0.875)$

7K

1 (b) $x^2 - 12x + k - 4 = 0$

$b^2 - 4ac = 0$ for real & equal roots.

$(-12)^2 - 4(1)(k-4) = 0$

$144 - 4(k-4) = 0$

$144 - 4k + 16 = 0$

$160 - 4k = 0$

$160 = 4k$

$\underline{\underline{k = 40}}$

1 (d) $kx^2 - 4x + k - 3 = 0$

$b^2 - 4ac = 0$ for real and equal roots

$(-4)^2 - 4(k)(k-3) = 0$

$16 - 4k(k-3) = 0$

$16 - 4k^2 + 12k = 0$

$0 = 4k^2 - 12k - 16$

$0 = 4(k^2 - 3k - 4)$

$0 = 4(k-4)(k+1)$

$k - 4 = 0, \quad k + 1 = 0$

$\underline{\underline{k = 4, \quad k = -1}}$

7K

2(a) $(2k+1)x^2 + 12x + k = 0$

$b^2 - 4ac = 0$ for equal roots

$(12)^2 - 4(2k+1)(k) = 0$

$144 - (8k+4)(k) = 0$

$144 - 8k^2 - 4k = 0$

$0 = 8k^2 + 4k - 144$

$0 = 4(2k^2 + k - 36)$

$0 = 4(2k+9)(k-4)$

$2k + 9 = 0 \qquad k - 4 = 0$

$2k = -9$

$\cancel{k = -\frac{9}{2}} \qquad k = 4$

as $k > 0$ $\quad k = 4$

(b)

sub in $k = 4$

$(2(4)+1)x^2 + 12x + 4 = 0$

$9x^2 + 12x + 4 = 0$

$(3x+2)(3x+2) = 0$

$3x + 2 = 0$

$3x = -2$

$x = -\frac{2}{3}$

7K

3 (b) $2x^2 + 5x - 3 \leq 0$

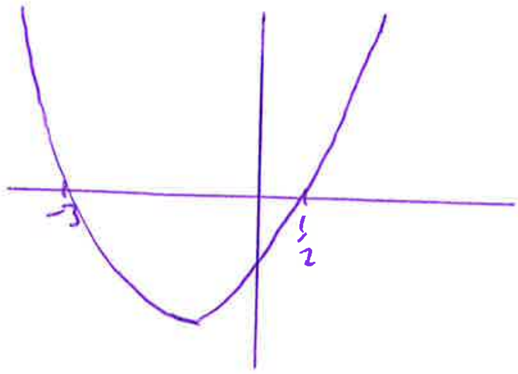
$2x^2 + 5x - 3 = 0$

$(2x - 1)(x + 3) = 0$

$2x - 1 = 0 \qquad x + 3 = 0$

$\underline{\underline{x = \frac{1}{2}}} \qquad \underline{\underline{x = -3}}$

• Solve equal to zero to find roots



• Sketch parabola

$\underline{\underline{-3 \leq x \leq \frac{1}{2}}}$

• State range of values of $x$ where $2x^2 + 5x - 3 \leq 0$ i.e. below the $x$-axis or on the $x$-axis

(d) $16 + 6x - x^2 < 0$

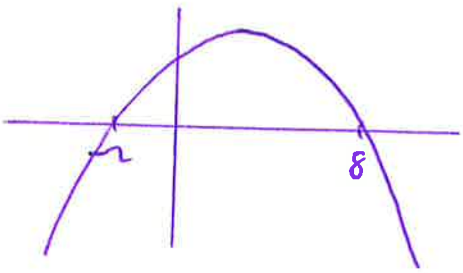
$16 + 6x - x^2 = 0$

$x^2 - 6x - 16 = 0$

$(x - 8)(x + 2) = 0$

$x = 8, \ x = -2$

• Find roots



$\underline{\underline{x < -2, \ x > 8}}$

7k

3 (f) $-6x^2 + 5x - 1 \le 0$

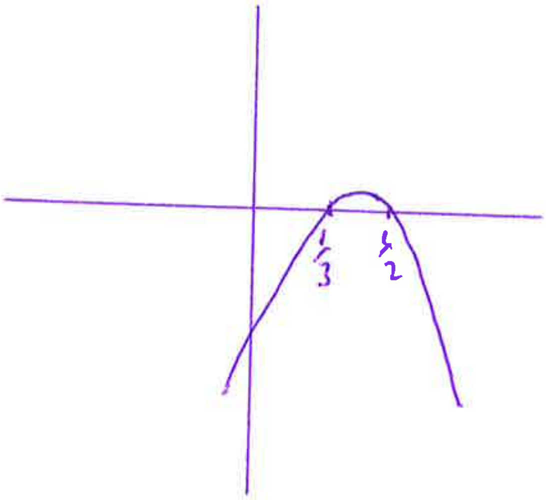
$-6x^2 + 5x - 1 = 0$ • Find roots

$0 = 6x^2 - 5x + 1$

$0 = (3x - 1)(2x - 1)$

$3x - 1 = 0 \qquad 2x - 1 = 0$

$x = \frac{1}{3} \qquad x = \frac{1}{2}$




$\underline{\underline{x \le \frac{1}{3}, \quad x \ge \frac{1}{2}}}$

7K

4(a) $3x^2 - 4x + 1 + k = 0$

$b^2 - 4ac \geq 0$ for real roots

• This statement include both real & equal and real & distinct roots

$(-4)^2 - 4(3)(1+k) \geq 0$

$16 - 12(1+k) \geq 0$

$16 - 12 - 12k \geq 0$

$4 - 12k \geq 0$

$4 \geq 12k$

$\frac{4}{12} \geq k$

$\underline{\underline{k \leq \frac{1}{3}}}$

4(c) $(k+1)x^2 - 4kx + k = 0$

$b^2 - 4ac \geq 0$ for real roots

$(-4k)^2 - 4(k+1)(k) \geq 0$

$16k^2 - (4k+4)(k) \geq 0$

$16k^2 - 4k^2 - 4k \geq 0$

$12k^2 - 4k \geq 0$

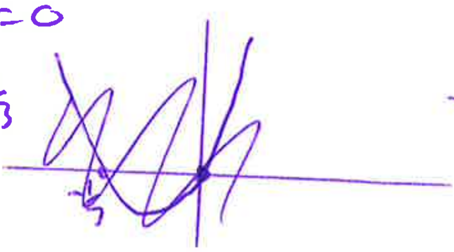
• quadratic inequality so much sketch graph to solve.

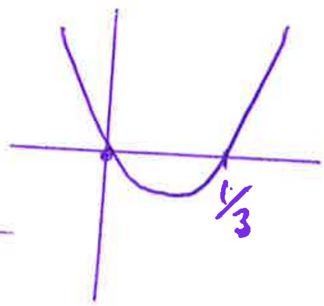
Roots: $12k^2 - 4k = 0$

$4(3k^2 - k) = 0$

$4k(3k - 1) = 0$

$k = 0 \quad k = +\frac{1}{3}$






$\underline{\underline{k \leq 0, \; k \geq \frac{1}{3}}}$

7K

(5) $x^2 + 6x + 9 + k = 0$

$b^2 - 4ac < 0$ for no real roots

$(6)^2 - 4(1)(9+k) < 0$

$36 - 4(9+k) < 0$

$36 - 36 - 4k < 0$

$-4k < 0$

$0 < 4k$

$0 < k$

$\underline{\underline{k > 0}}$

(6) $kx^2 - 4x + 3 + k = 0$

$b^2 - 4ac \geq 0$ for real roots

$(-4)^2 - 4(k)(3+k) \geq 0$

$16 - 4k(3+k) \geq 0$

$16 - 12k - 4k^2 \geq 0$ • Sketch to find values of k.

or ⇒

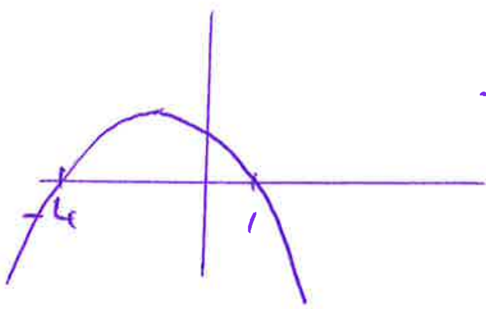
Roots: $16 - 12k - 4k^2 = 0$

$0 = 4k^2 + 12k - 16$

$0 = 4(k^2 + 3k - 4)$

$0 = 4(k+4)(k-1)$

$\underline{k = 1, \ k = -4}$




$\underline{\underline{-4 \leq k \leq 1}}$

7/7

8 $f(x) = g(x)$

$2x^2 - kx + 5 = x^2 - 3kx + 5 - 4k$

$x^2 + 2kx + 4k = 0$

$b^2 - 4ac > 0$ for real & distinct roots.

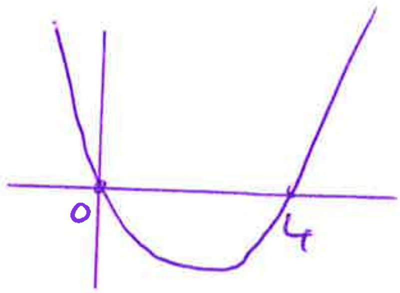
$(2k)^2 - 4(1)(4k) > 0$

$4k^2 - 16k > 0$

Roots $4k^2 - 16k = 0$

$4k(k-4) = 0$

$k=0 \quad k=4$




$k < 0, \quad k > 4$

7.~~K~~

9 (a)

$$\begin{array}{r|rrrr} 2 & 3 & a & -19 & 6 \\ & & 6 & 2a+12 & 4a-14 \\ \hline & 3 & a+6 & 2a-7 & \boxed{4a-8} \end{array} = 0 \quad \text{as } x=2 \text{ is a root.}$$

$4a - 8 = 0$

$4a = 8$

$\underline{\underline{a = 2}}$

sub $a = 2 \Rightarrow y = (x-2)(3x^2 + 8x - 3)$

(b) $y = y$

$3x^3 + 2x^2 - 19x + 6 = x^2 - 25x + 16$

$3x^3 + \cancel{2}x^2 + 6x - 10 = 0$

$f(1) = 0$

$$\begin{array}{r|rrrr} 1 & 3 & 1 & 6 & -10 \\ & & 3 & 4 & 10 \\ \hline & 3 & 4 & 10 & \boxed{0} \end{array}$$

as $f(1) = 0$ $x = 1$ is a root and $(x-1)$ is a factor.

$3x^3 + x^2 + 6x - 10 = 0$

$(x-1)(3x^2 + 4x + 10) = 0$

consider $3x^2 + 4x + 10 = 0$

$b^2 - 4ac$

$= (4)^2 - 4(3)(10)$

$= -104$

$b^2 - 4ac = 0 \therefore 3x^2 + 4x + 10 = 0$ has no real roots and the only point of contact will be at $x = 1$

$y(1) = (1)^2 - 25(1) + 16$

$= -8$

$\underline{\underline{(1, -8)}}$